Schutz 8.4 (a)

(i) Some quick tests

$$(I - A + A^2 - A^3 + A^4 - \dots)_{ij} = \delta_{ij} - A_{ij} + A_{ik}A_{kj} - A_{ik}A_{kl}A_{lj} + \dots$$

Test absolute convergence, sum the abs. val. of the terms:

$$|\delta_{ij}| + |A_{ij}| + |A_{ik}A_{kj}| + |A_{ik}A_{kl}A_{lj}| + \dots$$

$$\leq 1 + \underset{\Uparrow \atop \ll \frac{1}{n}}{|A_{ij}|} + \underset{\Uparrow \atop \ll \frac{1}{n}.}{|A_{ik}|}|A_{kj}| + |A_{ik}||A_{kl}||A_{lj}|.$$

$$|\delta_{ij}| + |A_{ij}| + |A_{ik}A_{kj}| + |A_{ik}A_{kl}A_{lj}| + \dots$$

$$\underset{1}{\Downarrow} \qquad \underset{\ll 1}{\Downarrow} \qquad (\ll \tfrac{1}{n})^2 n \;\; (\ll \tfrac{1}{n})(\ll 1) \qquad \Uparrow \;\; (\ll \tfrac{1}{n})(\ll \tfrac{1}{n})(\ll \tfrac{1}{n}) \tfrac{1}{n^2} \;\; (\ll \tfrac{1}{n})(\ll 1)^2$$

$$= 1 + (\ll \tfrac{1}{n}) + (\ll 1)(\ll \tfrac{1}{n}) + (\ll \tfrac{1}{n})(\ll 1)^2.$$

$$= 1 + (\ll \tfrac{1}{n})\left[1 + (\ll 1) + (\ll 1)^2 + \dots\right]$$

$$= 1 + (\ll \tfrac{1}{n})\frac{1}{1-(\ll 1)} = \boxed{1 + \frac{1}{n}\,\frac{\ll 1}{1-(\ll 1)}}$$

I'm not proud of this argument.

(iii) $(I+A)[I-A+A^2-A^3+A^4-\dots]$

$$= I - A + A^2 - A^3 + A^4 \dots + A - A^2 + A^3 - A^4 \dots$$

$$= \boxed{I}$$

A less rigorous way: treat A as a #:

$$(1+A)^{-1} = \frac{1}{1+A} = \frac{1}{1-(-A)}$$

$$= \sum_{n=0}^{\infty} (-A)^n.$$

(b) $\Lambda^{\alpha'}{}_{\beta} = \frac{\partial x^{\alpha'}}{\partial x^{\beta}} = \delta^{\alpha'}{}_{\beta} + \xi^{\alpha}{}_{,\beta}$

$\Lambda^{\alpha'}{}_{\beta} : x^{\beta} \to x^{\alpha'}$

$\Lambda^{\beta}{}_{\alpha'} : x^{\alpha'} \to x^{\beta} = (\Lambda^{\alpha'}{}_{\beta})^{-1}$

$$\Rightarrow \Lambda^{\beta}{}_{\alpha'} = \delta^{\beta}{}_{\alpha} - \xi^{\beta}{}_{,\alpha} + O(\xi^2)$$

$$\Rightarrow \boxed{\Lambda^{\alpha}{}_{\beta'} = \delta^{\alpha}{}_{\beta} - \xi^{\alpha}{}_{,\beta} + O(\xi^2)}$$

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